CS 33

Data Representation (Part 3)

Byte Ordering

- **Four-byte integer**
  - 0x76543210
- **Stored at location 0x100**
  - which byte is at 0x100?
  - which byte is at 0x103?

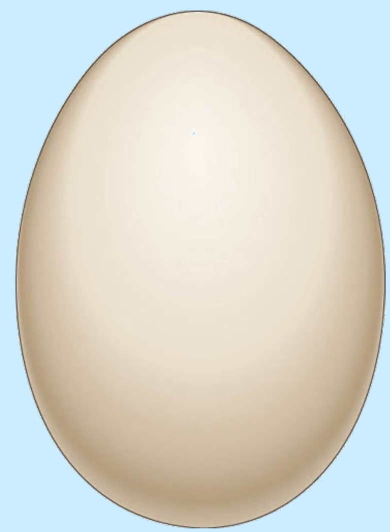
?

10	32	54	76
0x100	0x101	0x102	0x103

Little-endian

76	54	32	10
0x100	0x101	0x102	0x103

Big-endian



Which Byte Ordering Do We Use?

```
int main() {
    unsigned int x = 0x03020100;
    unsigned char *xarray = (unsigned char *)&x;
    for (int i=0; i<4; i++) {
        printf("%02x", xarray[i]);
    }
    printf("\n");
    return 0;
}
```

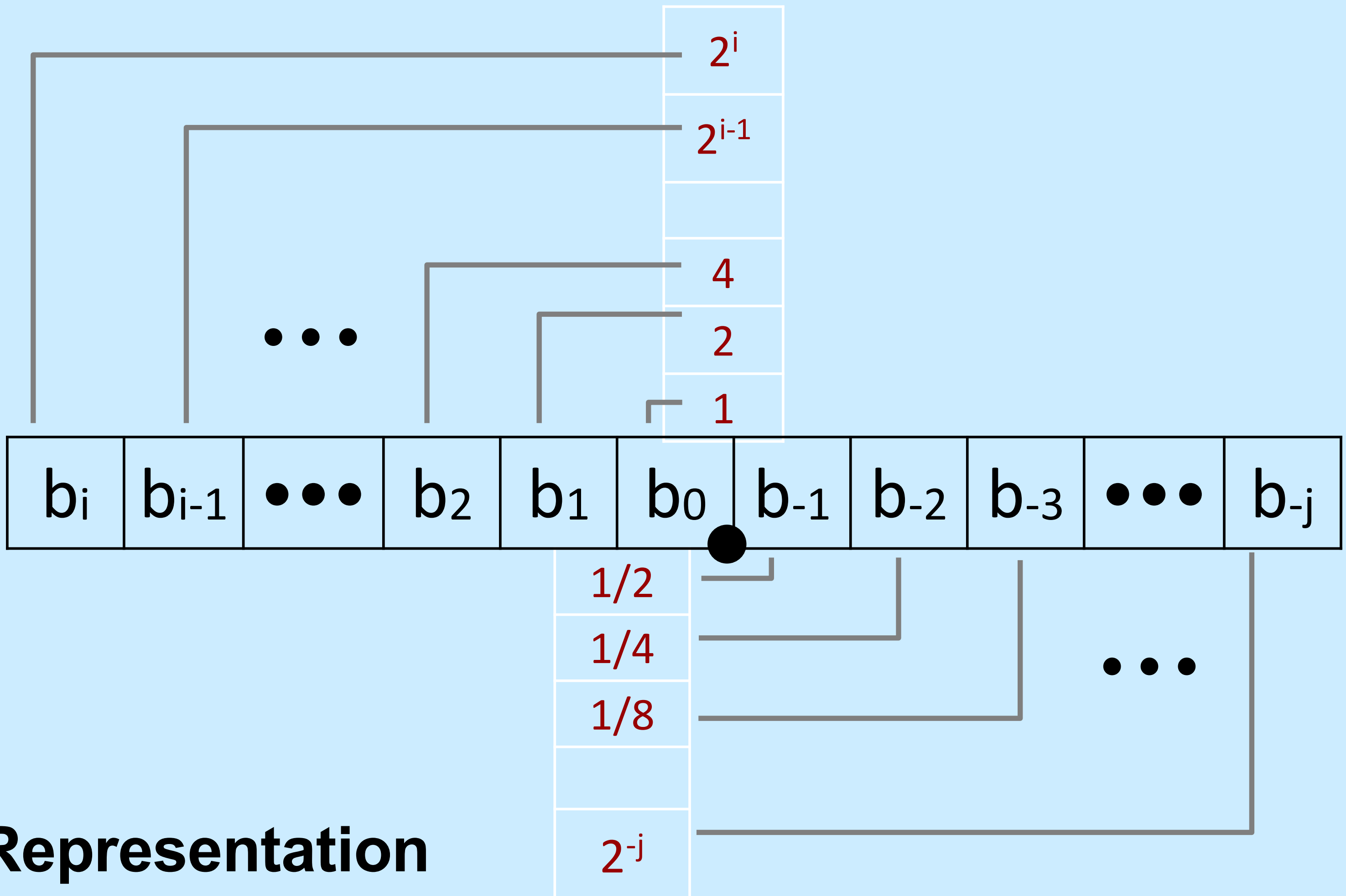
Possible results:

```
00010203
03020100
```

Fractional binary numbers

- What is 1011.101_2?

Fractional Binary Numbers



- **Representation**
  - **bits to right of "binary point" represent fractional powers of 2**
  - **represents rational number:** $$\sum_{k=-j}^{i} b_k \times 2^k$$

Representable Numbers

- **Limitation #1**
  - can exactly represent only numbers of the form $n/2^k$
    - » other rational numbers have repeating bit representations
  - value representation
    - » 1/3 `0.0101010101[01]…`$_2$
    - » 1/5 `0.001100110011[0011]…`$_2$
    - » 1/10 `0.0001100110011[0011]…`$_2$

- **Limitation #2**
  - just one setting of decimal point within the *w* bits
    - » limited range of numbers (very small values? very large?)

IEEE Floating Point

- IEEE Standard 754
  - established in 1985 as uniform standard for floating point arithmetic
    - before that, many idiosyncratic formats
  - supported on all major CPUs
- Driven by numerical concerns
  - nice standards for rounding, overflow, underflow
  - hard to make fast in hardware
    - numerical analysts predominated over hardware designers in defining standard

Floating-Point Representation

- **Numerical Form:**

$$(-1)^s \; M \;\; 2^E$$

  - sign bit **s** determines whether number is negative or positive
  - significand **M** normally a fractional value in range [1.0,2.0)
  - exponent **E** weights value by power of two
- **Encoding**
  - MSB `s` is sign bit **s**
  - `exp` field encodes **E** (but is not equal to E)
  - `frac` field encodes **M** (but is not equal to M)

Precision options

- **Single precision: 32 bits**

s	exp	frac
1	8-bits	23-bits

- **Double precision: 64 bits**

s	exp	frac
1	11-bits	52-bits

- **Extended precision: 80 bits (Intel only)**

s	exp	frac
1	15-bits	64-bits

"Normalized" Values

- When: exp ≠ 000…0 and exp ≠ 111…1
- Exponent coded as biased value: E = Exp – Bias
  - exp: unsigned value exp
  - bias = 2^{k-1} - 1, where k is number of exponent bits
    - » single precision: 127 (Exp: 1…254, E: -126…127)
    - » double precision: 1023 (Exp: 1…2046, E: -1022…1023)
- Significand coded with implied leading 1: M = $1.xxx…x_2$
  - xxx…x: bits of frac
  - minimum when frac=000…0 (M = 1.0)
  - maximum when frac=111…1 (M = 2.0 – ε)
  - get extra leading bit for "free"

Normalized Encoding Example

- **Value:** `float F = 15213.0;`
  - $15213_{10} = 11101101101101_2$
    $= 1.1101101101101_2 \times 2^{13}$

- **Significand**

  M = `1.1101101101101`$_2$

  `frac` = `11011011011010000000000`$_2$

- **Exponent**

  E = **13**

  bias = **127**

  exp = **140** = `10001100`$_2$

- **Result:**

s	exp	frac
0	10001100	11011011011010000000000

Denormalized Values

- Condition: exp = `000…0`
- Exponent value: `E` = –Bias + 1 (instead of `E` = 0 – Bias)
- Significand coded with implied leading 0:
  $M = 0.xxx\ldots x_2$
  - `xxx…x`: bits of `frac`
- Cases
  - `exp` = `000…0`, `frac` = `000…0`
    - » represents zero value
    - » note distinct values: +0 and –0 (why?)
  - `exp` = `000…0`, `frac` ≠ `000…0`
    - » numbers closest to 0.0
    - » equispaced

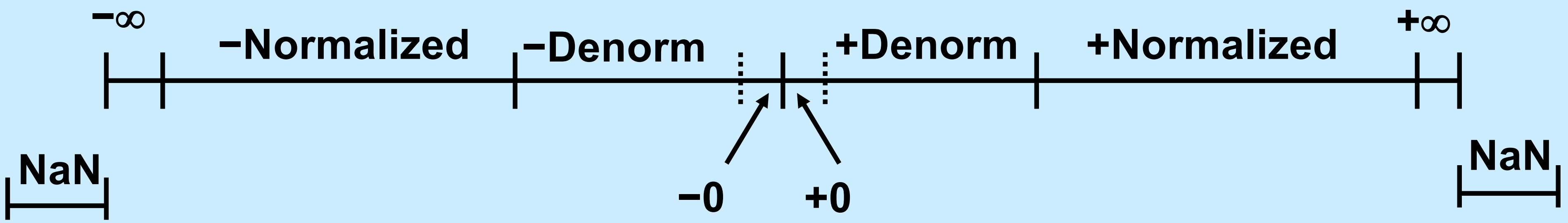
Special Values

- Condition: `exp` = 111…1

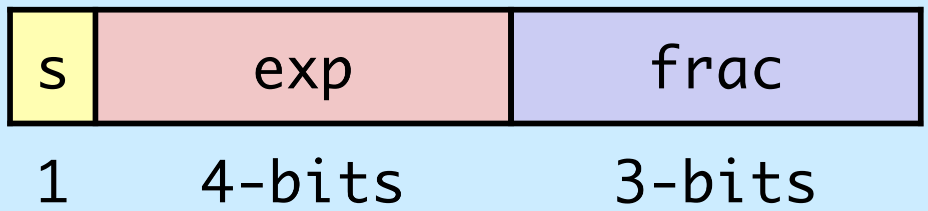
- Case: `exp` = 111…1, `frac` = 000…0
  - represents value ∞ (infinity)
  - operation that overflows
  - both positive and negative
  - e.g., 1.0/0.0 = −1.0/−0.0 = +∞, 1.0/−0.0 = −∞

- Case: `exp` = 111…1, `frac` ≠ 000…0
  - not-a-number (NaN)
  - represents case when no numeric value can be determined
  - e.g., sqrt(–1), ∞ − ∞, ∞ × 0

Visualization: Floating-Point Encodings



Tiny Floating-Point Example



- **8-bit Floating Point Representation**
  - the sign bit is in the most significant bit
  - the next four bits are the exponent, with a bias of 7
  - the last three bits are the `frac`

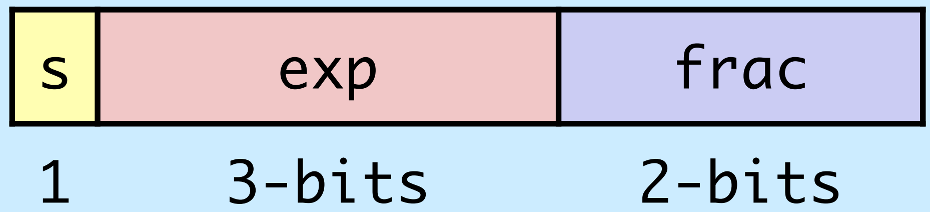
- **Same general form as IEEE Format**
  - normalized, denormalized
  - representation of 0, NaN, infinity

Dynamic Range (Positive Only)

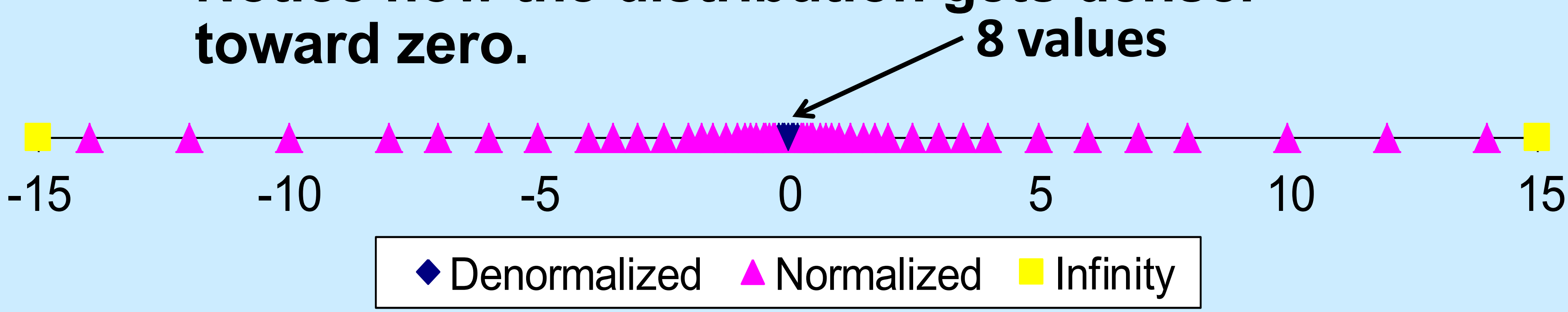
	s	exp	frac	E	Value	
Denormalized numbers	0	0000	000	-6	0	
	0	0000	001	-6	1/8*1/64 = 1/512	closest to zero
	0	0000	010	-6	2/8*1/64 = 2/512	
	…					
	0	0000	110	-6	6/8*1/64 = 6/512	
	0	0000	111	-6	7/8*1/64 = 7/512	largest denorm
Normalized numbers	0	0001	000	-6	8/8*1/64 = 8/512	smallest norm
	0	0001	001	-6	9/8*1/64 = 9/512	
	…					
	0	0110	110	-1	14/8*1/2 = 14/16	
	0	0110	111	-1	15/8*1/2 = 15/16	closest to 1 below
	0	0111	000	0	8/8*1 = 1	
	0	0111	001	0	9/8*1 = 9/8	closest to 1 above
	0	0111	010	0	10/8*1 = 10/8	
	…					
	0	1110	110	7	14/8*128 = 224	
	0	1110	111	7	15/8*128 = 240	largest norm
	0	1111	000	n/a	inf	

Distribution of Values

- **6-bit IEEE-like format**
  - **e = 3 exponent bits**
  - **f = 2 fraction bits**
  - **bias is $2^{3-1}-1 = 3$**

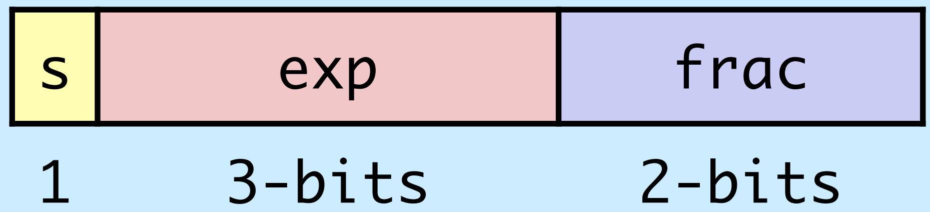


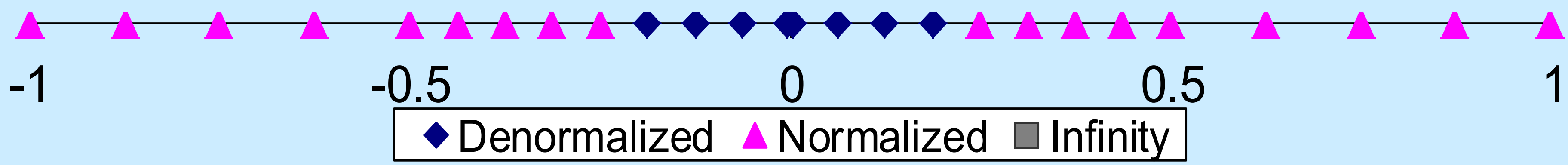
- **Notice how the distribution gets denser toward zero.**



Distribution of Values (close-up view)

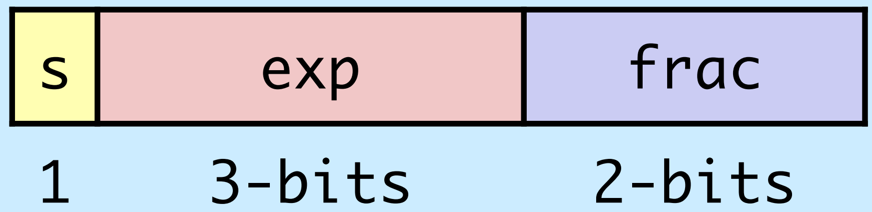
- **6-bit IEEE-like format**
  - **e = 3 exponent bits**
  - **f = 2 fraction bits**
  - **bias is 3**





Quiz 1

- **6-bit IEEE-like format**
  - e = 3 exponent bits
  - f = 2 fraction bits
  - bias is 3



What number is represented by 0 010 10?

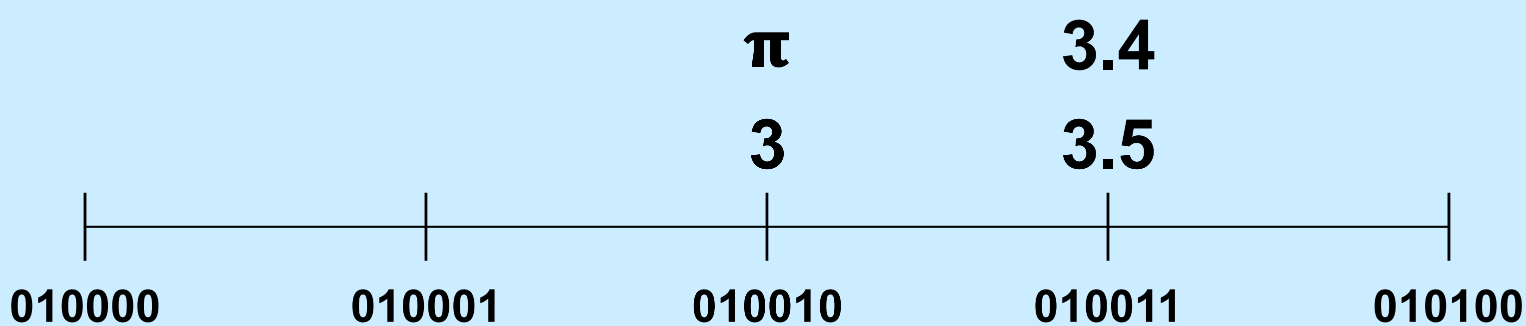
a) 3
b) 1.5
c) .75
d) none of the above

Mapping Real Numbers to Float

- The real number 3 is represented as
  0 100 10
- The real number 3.5 is represented as
  0 100 11
- How is the real number 3.4 represented?

  0 100 11
- How is the real number π represented?

  0 100 10

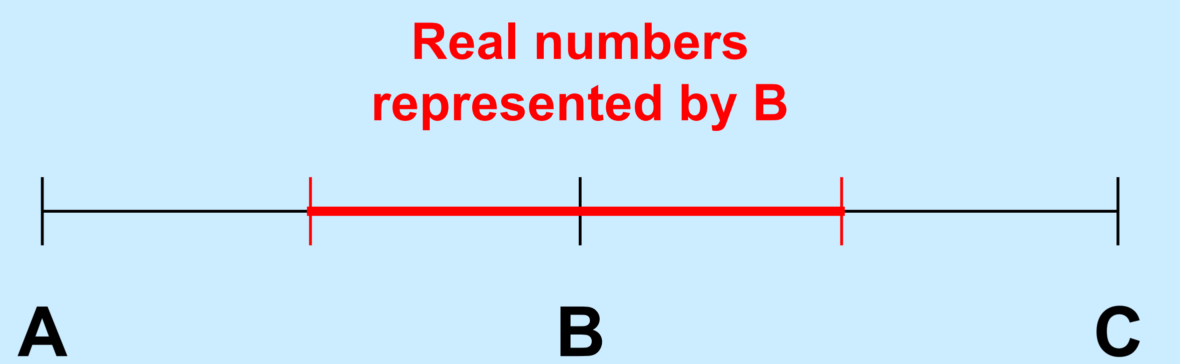


Mapping Real Numbers to Float

- If R is a real number, it's mapped to the floating-point number whose value is closest to R
- What if it's midway between two values?
  - rounding rules determine outcome

Floats are Sets of Values

- If A, B, and C are successive floating-point values
  - e.g., 010001, 010010, and 010011
- B represents all real numbers from midway between A and B through midway between B and C



Significance

- **Normalized numbers**
  - for a particular exponent value E and an S-bit significand, the range from 2^E up to 2^{E+1} is divided into 2^S equi-spaced floating-point values
    - » thus each floating-point value represents $1/2^S$ of the range of values with that exponent
    - » all bits of the significand are important
    - » we say that there are S significant bits – for reasonably large S, each floating-point value covers a rather small part of the range
      - high accuracy
      - for S=23 (32-bit float), accurate to one in 2^{23} (.0000119% accuracy)

Significance

- **Unnormalized numbers**
  - high-order zero bits of the significand aren't important
  - in 8-bit floating point, 0 0000 001 represents 2^{-9}
    - » it is the only value with that exponent: 1 significant bit (either 2^{-9} or 0)
    - » 50% accuracy
  - 0 0000 010 represents 2^{-8}
    0 0000 011 represents $1.5*2^{-8}$
    - » only two values with exponent -8: 2 significant bits (encoding those two values, as well as 2^{-9} and 0)
    - » 25% accuracy
  - fewer significant bits means less accuracy
  - 0 0000 001 represents a range of values from $.5*2^{-9}$ to $1.5*2^{-9}$

+/− Zero

- Only one zero for ints
  - an int is a single number, not a range of numbers, thus there can be only zero
- Floating-point zero
  - a range of numbers around the real 0
  - it really matters which side of 0 we're on!
    - » a very large negative number divided by a very small negative number should be positive

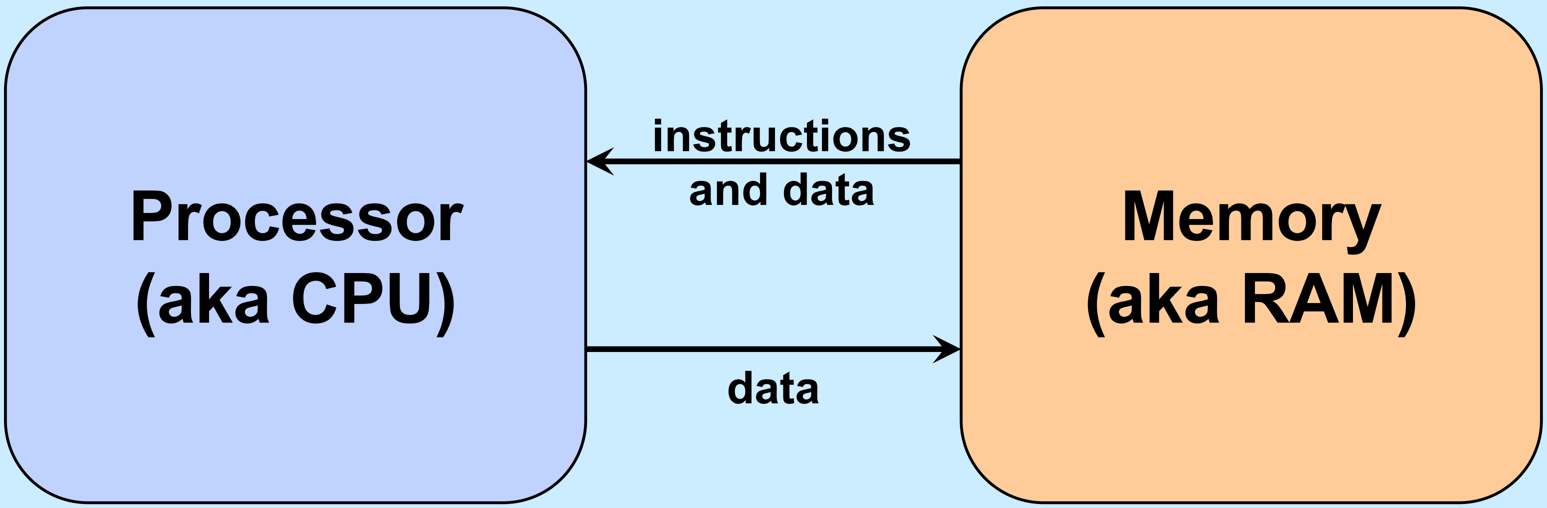
      $-\infty/-0 = +\infty$
    - » a very large positive number divided by a very small negative number should be negative

      $+\infty\ /-0 = -\infty$

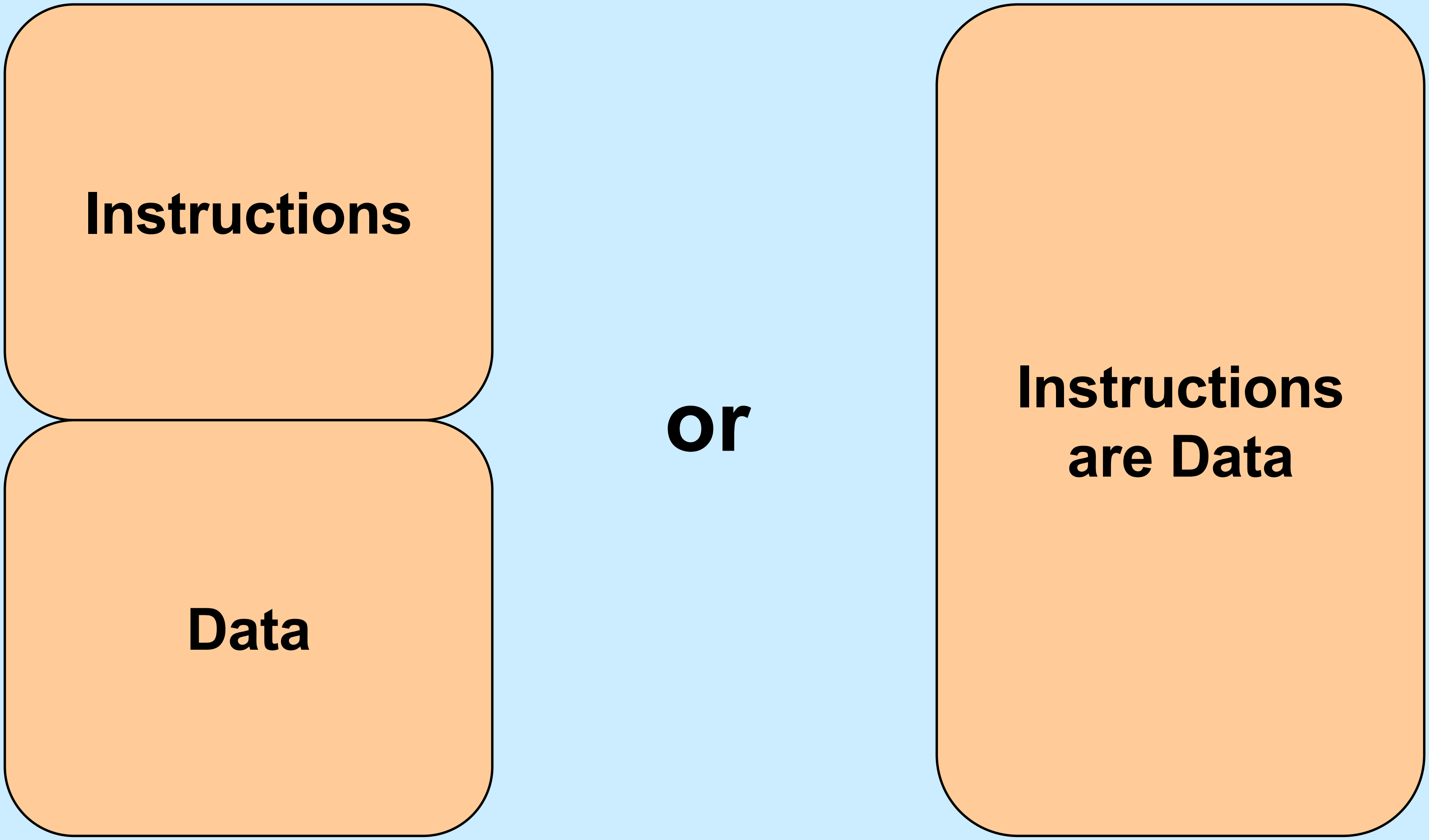
CS 33

Intro to Machine Programming

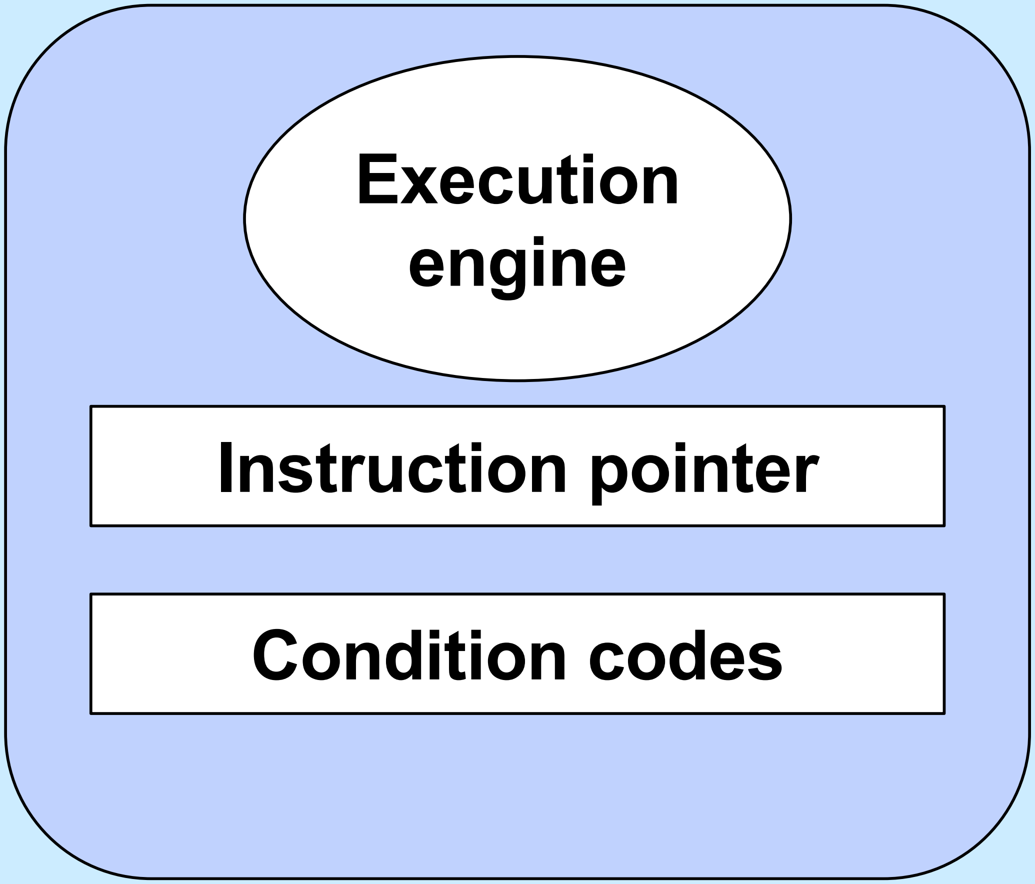
Machine Model



Memory



Processor: Some Details



Processor: Basic Operation

```
while (forever) {
  fetch instruction IP points at
  decode instruction
  fetch operands
  execute
  store results
  update IP and condition code
}
```

Instructions ...

Op code	Operand1	Operand2	...

Operands

- Form
  - immediate vs. reference
    - value vs. address
- How many?
  - 3
    - add a,b,c
      - c = a + b
  - 2
    - add a,b
      - b += a

Operands (continued)

- **Accumulator**
  - special memory in the processor
    - » known as a *register*
    - » fast access
  - allows single-operand instructions
    - » add a
      - acc += a
    - » add b
      - acc += b

From C to Assembler ...

```
a = (b + c) * d;

    mov   b,%acc
    add   c,%acc
    mul   d,%acc
    mov   %acc,a
```

```
if (a<b)
   c = 1;
else
   d = 1;

   cmp   a,b
   jge   .L1
   mov   $1,c
   jmp   .L2
.L1
   mov   $1,d
.L2
```

immediate operand (the circled $1 in `mov $1,c`)

immediate operand (the circled $1 in `mov $1,d`)

Condition Codes

- **Set of flags giving status of most recent operation:**
  - zero flag
    - » result was zero
  - sign flag
    - » for signed arithmetic interpretation: sign bit is set
  - overflow flag
    - » for signed arithmetic interpretation
  - carry flag (generated by carry or borrow out of most-significant bit)
    - » for unsigned arithmetic interpretation
- **Set implicitly by arithmetic instructions**
- **Set explicitly by compare instruction**
  - cmp a,b
    - » sets flags based on result of b-a

Examples (1)

- **Assume 32-bit arithmetic**

- **x is `0x80000000`**
  - **TMIN if interpreted as two's-complement**
  - **2^{31} if interpreted as unsigned**
- **x-1 (`0x7fffffff`)**
  - **TMAX if interpreted as two's-complement**
  - **2^{31}-1 if interpreted as unsigned**
  - **zero flag is not set**
  - **sign flag is not set**
  - **overflow flag is set**
  - **carry flag is not set**

Examples (2)

- **x is `0xffffffff`**
  - **-1 if interpreted as two's-complement**
  - **UMAX (2^{32}-1) if interpreted as unsigned**
- **x+1 (`0x00000000`)**
  - **zero under either interpretation**
  - **zero flag is set**
  - **sign flag is not set**
  - **overflow flag is not set**
  - **carry flag is set**

Examples (3)

- **x is `0xffffffff`**
  - -1 if interpreted as two's-complement
  - UMAX (2^{32}-1) if interpreted as unsigned
- **x+2 (`0x00000001`)**
  - (+)1 under either interpretation
  - zero flag is not set
  - sign flag is not set
  - overflow flag is not set
  - carry flag is set

Quiz 2

- Set of flags giving status of most recent operation:
  - zero flag
    - » result was zero
  - sign flag
    - » for signed arithmetic interpretation: sign bit is set
  - overflow flag
    - » for signed arithmetic interpretation
  - carry flag (generated by carry or borrow out of most-significant bit)
    - » for unsigned arithmetic interpretation
- Set explicitly by compare instruction
  - cmp a,b
    - » sets flags based on result of b-a

Which flags are set to one by “cmp $2,$1”?

a) overflow flag only
b) carry flag only
c) sign and carry flags only
d) sign and overflow flags only
e) sign, overflow, and carry flags

Jump Instructions

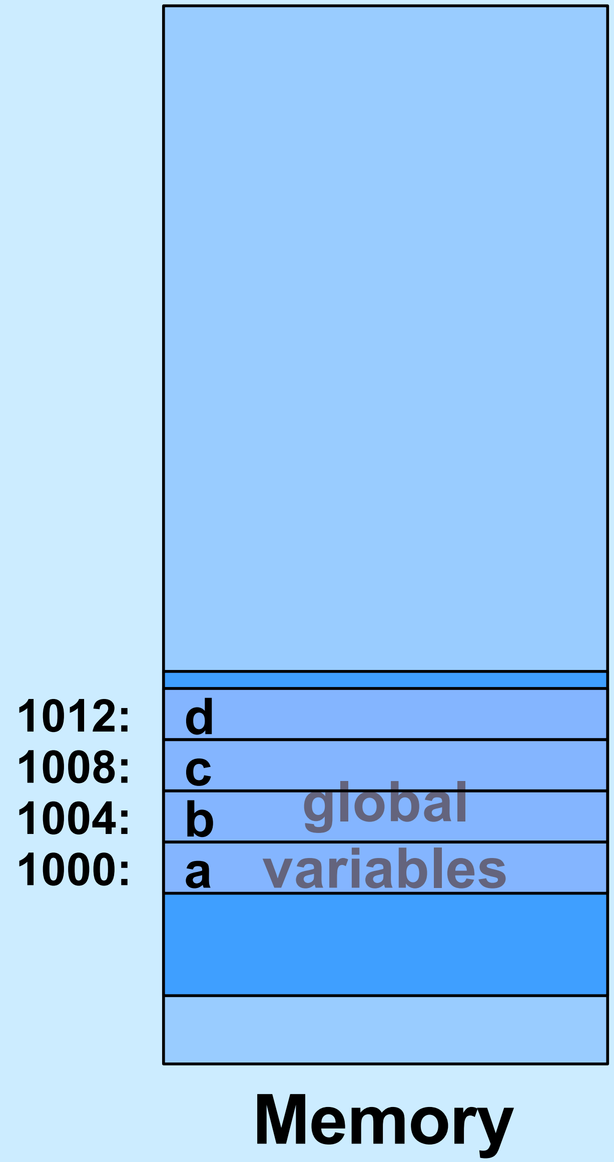
- **Unconditional jump**
  - just do it
- **Conditional jump**
  - to jump or not to jump determined by condition-code flags
  - field in the op code indicates how this is computed
  - in assembler language, simply say
    - » je
      - jump on equal
    - » jne
      - jump on not equal
    - » jg
      - jump on greater than (signed)
    - » etc.

Addresses

```
int a, b, c, d;

int main() {
    a = (b + c) * d;
    ...
}
```

```
mov    b,%acc
add    c,%acc
mul    d,%acc
mov    %acc,a
```

```
mov    1004,%acc
add    1008,%acc
mul    1012,%acc
mov    %acc,1000
```



Addresses

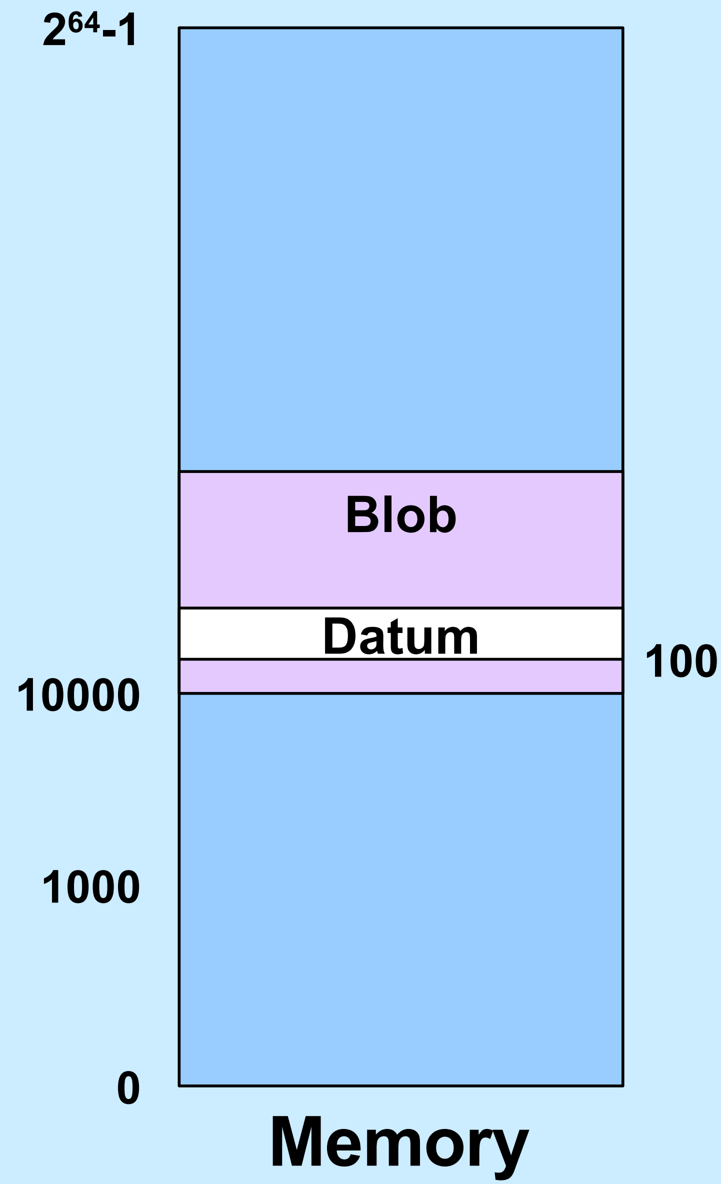
```
int b;

int func(int c, int d) {
    int a;
    a = (b + c) * d;
    ...
}

    mov    ?,%acc
    add    ?,%acc
    mul    ?,%acc
    mov    %acc,?
```

- One copy of *b* for duration of program’s execution
  - *b*’s address is the same for each call to *func*
- Different copies of *a*, *c*, and *d* for each call to *func*
  - addresses are different in each call

Relative Addresses

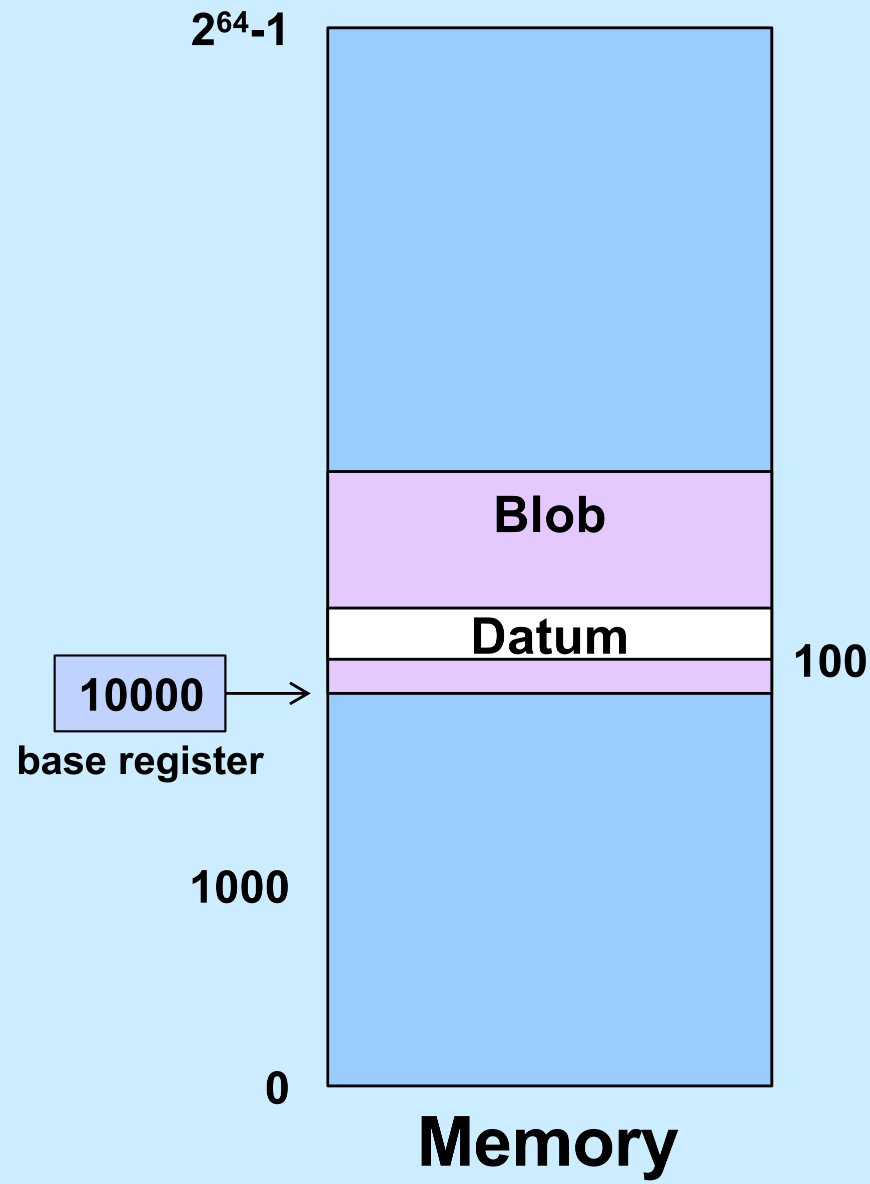
- **Absolute address**
  – actual location in memory
- **Relative address**
  – offset from some other location

- Blob's absolute address is 10000
- Datum's relative address (to Blob) is 100
  – its absolute address is 10100



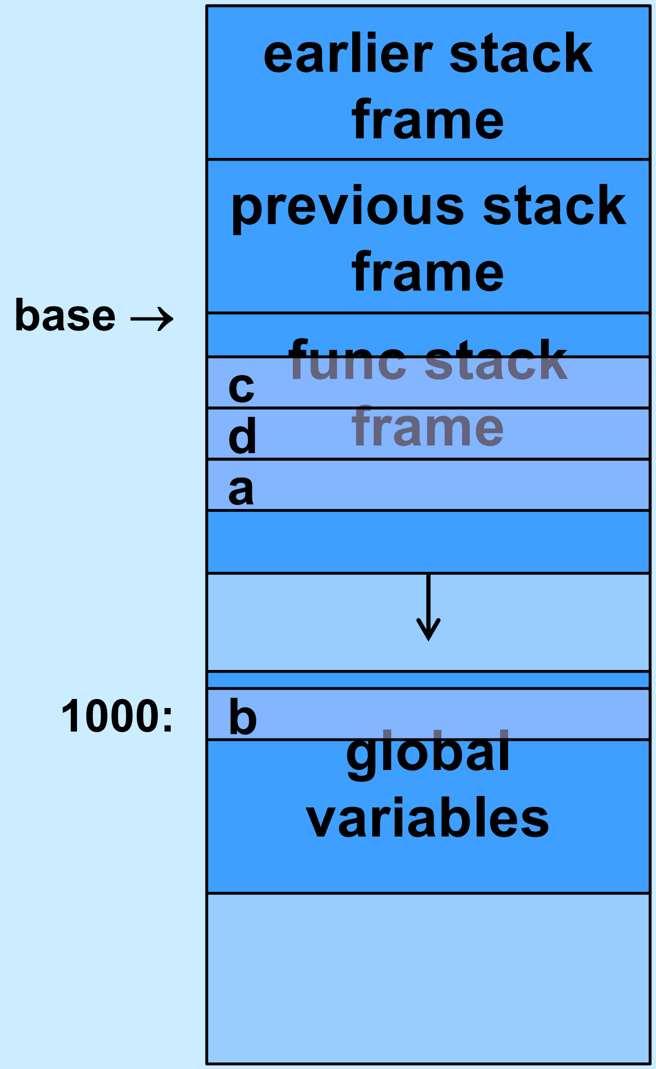
Base Registers

```
mov $10000, %base
mov $10, 100(%base)
```



Addresses

```
long b;

int func(long c, long d) {
    long a;
    a = (b + c) * d;
    ...
}

    mov   1000,%acc
    add   -8(%base),%acc
    mul   -16(%base),%acc
    mov   %acc,-24(%base)
```



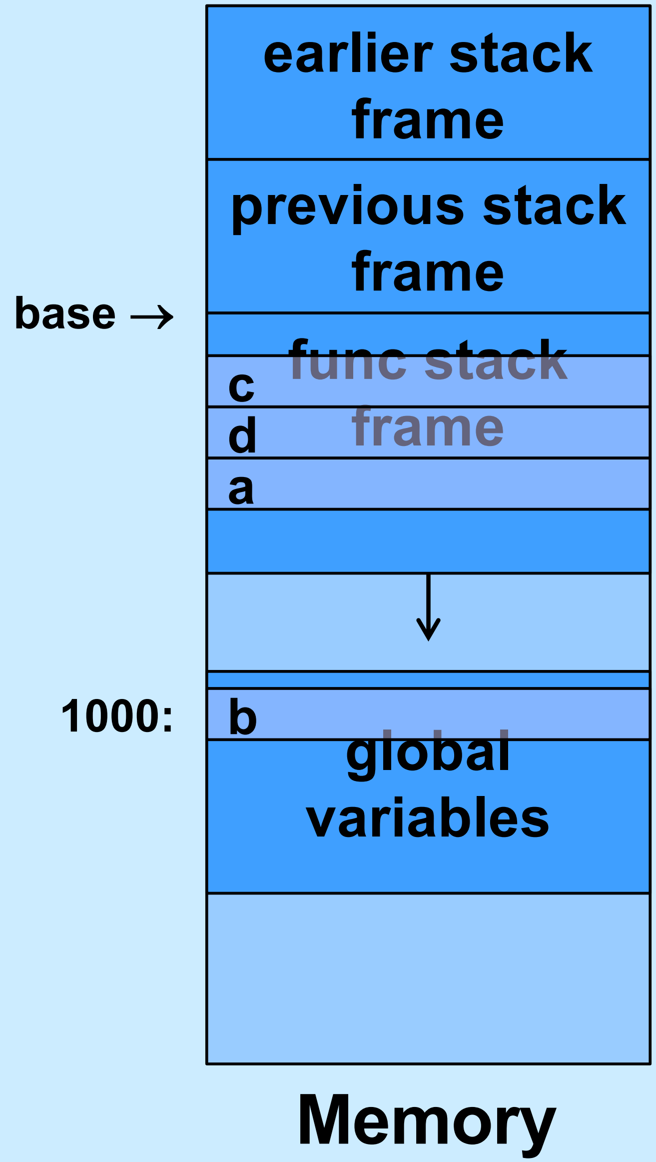
Memory

Quiz 3

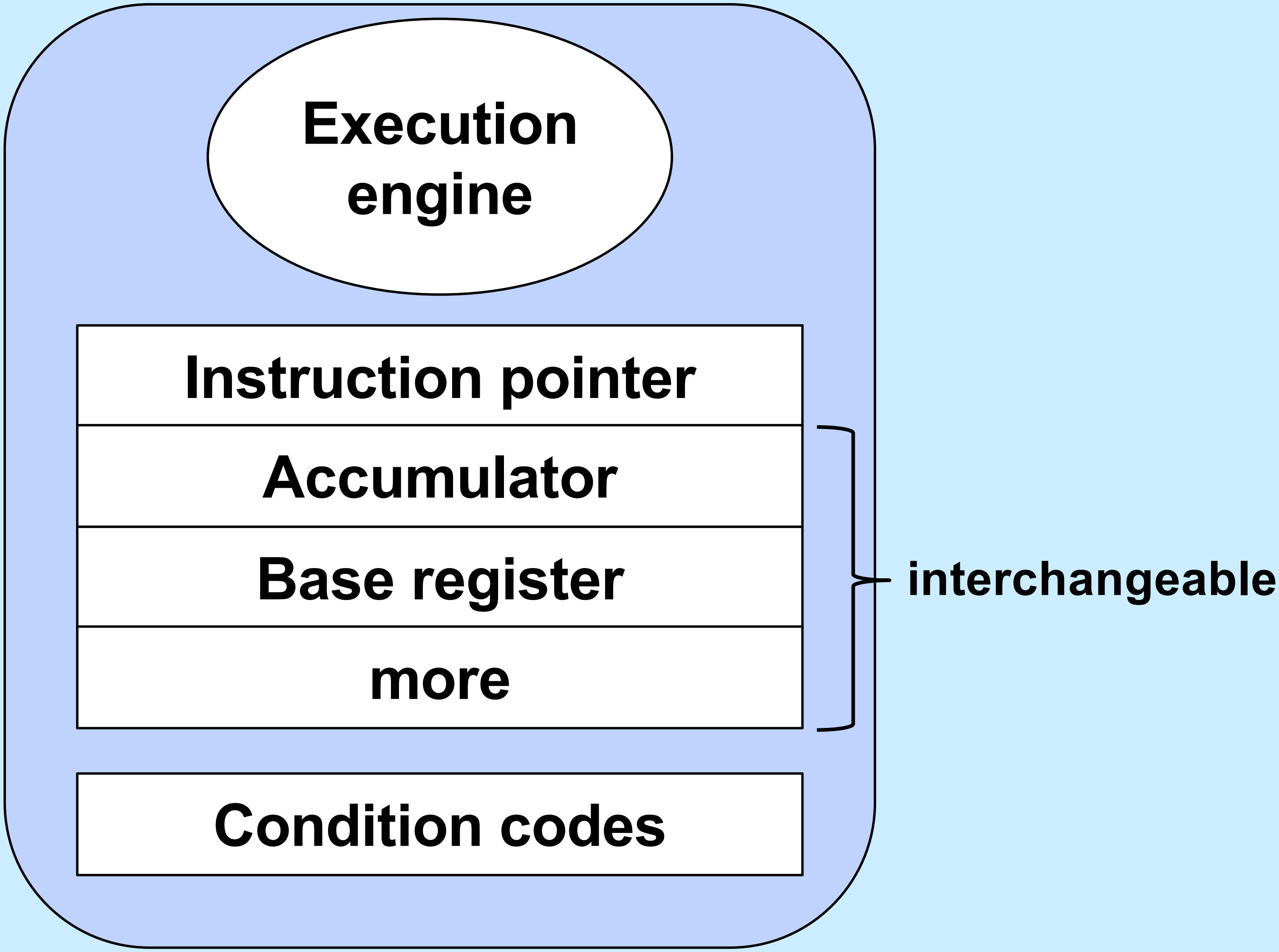
Suppose the value in *base* is 10,000. What is the address of *c*?

a) 10,016
b) 10,008
c) 9992
d) 9984

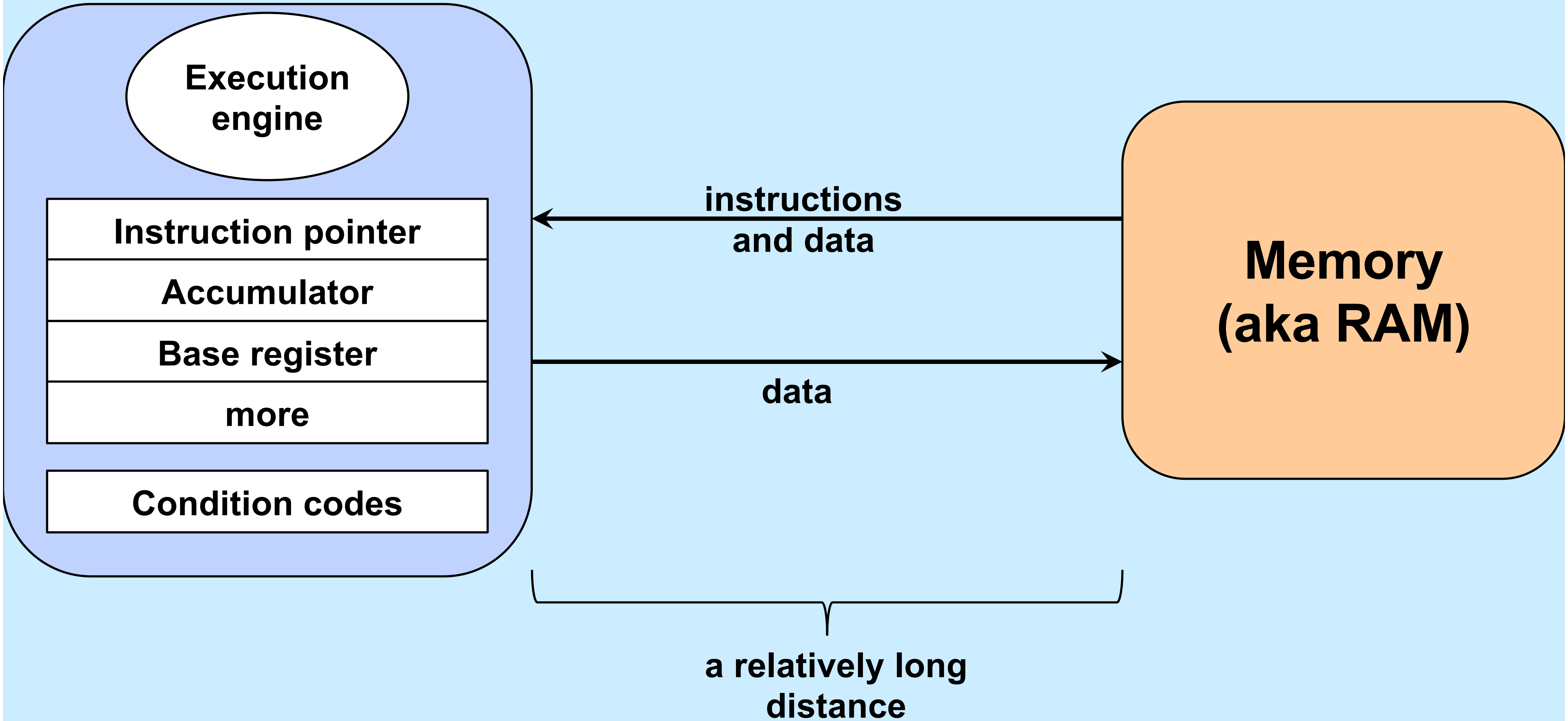
```
mov    1000,%acc
add    -8(%base),%acc
mul    -12(%base),%acc
mov    %acc,-16(%base)
```



Registers


Execution engine
Instruction pointer
Accumulator
Base register
more
interchangeable
Condition codes

Registers vs. Memory



Intel x86

- Intel created the 8008 (in 1972)
- 8008 begat 8080
- 8080 begat 8086
- 8086 begat 8088
- 8086 begat 286
- 286 begat 386
- 386 begat 486
- 486 begat Pentium
- Pentium begat Pentium Pro
- Pentium Pro begat Pentium II
- ad infinitum

IA32

2^{64}

- 2^{32} used to be considered a large number
  - one couldn't afford 2^{32} bytes of memory, so no problem with that as an upper bound
- Intel (and others) saw need for machines with 64-bit addresses
  - devised IA64 architecture with HP
    - » became known as Itanium
    - » very different from x86
- AMD also saw such a need
  - developed 64-bit extension to x86, called x86-64
- Itanium flopped
- x86-64 dominated
- Intel, reluctantly, adopted x86-64

Why Intel?

- Most CS Department machines are Intel
- An increasing number of personal machines are not
  - Apple has switched to ARM
  - packaged into their M1, M2, etc. chips
    - » "Apple Silicon"
- Intel x86-64 is very different from ARM64 — internally
- Programming concepts are similar
- We cover Intel; most of the concepts apply to ARM